1. [Start of transcript. Skip to the end.](https://courses.edx.org/xblock/block-v1:ColumbiaX+CSMM.101x+3T2020+type@vertical+block@f6cb826cba25408ba3a5a03231dd299a?show_title=0&show_bookmark_button=0#transcript-end-84ba0101e14a4a14b63c123a0d48bbd5)
2. It turned out that searching for a proof using search algorithms
3. is more efficient than doing model checking by integrating
4. all possible truth values for the propositions.
5. So the main reason is that, first of all,
6. truth tables are exponentials, and the number of propositions.
7. And furthermore, when we do inference,
8. we can simply ignore irrelevant information.
9. So remember, for example, in r3, r3
10. was not needed to derive any of the goals I wanted to check,
11. for which I was to check entailments, right?
12. So r3 was ignored altogether, and we don't
13. need to use it for inference.
14. However, when you do the model checking,
15. you have to consider all possible propositions
16. and all possible rules within your knowledge base.
17. So the idea of inference is simply
18. to repeat inference rules until you reach the goal.
19. Inference can be applied whenever suitable premises are
20. found in the knowledge base.
21. I'm going to keep going-- imply inference rules
22. as I did in the previous example.
23. So an important question is whether inference
24. is sound and complete.
25. So for soundess, to check whether the inference is sound,
26. it's enough to check that it is sound at each step
27. of the inference.
28. So to use the inference, we're going
29. to use the different proposition that we--
30. properties or proposition we have seen earlier.
31. For example, if you want to use modus ponens,
32. pa and p plus q infers q.
33. So this one can be expressed as p, and p implies q, implies q.
34. If you do the truth table of this proposition here,
35. you would find that it is a tautology, right?
36. So you find it true no matter what p and q are.
37. So we have a sound inference rule
38. that we are going to use over and over again,
39. along with other rules, such as the double implication
40. or the bi-directionals.
41. So anything that's p equivalent to q,
42. you'll find p implies q and q implies p.
43. You could also prove that this part implies
44. this part-- is a tautology.
45. So whenever I use a succession or an iteration
46. of sound inference rules, your process will be sound overall.
47. So how about completeness?
48. Completeness is actually trickier.
49. And the reason is that for soundess, it's
50. easy to show that old provable propositions about inference
51. are true.
52. This is because we're using to sound rule each time.
53. So we are only getting entailed formulas.
54. However, for completeness, what you want to prove
55. is that all true propositions are there are provable.
56. So this makes the knowledge representation larger.
57. So we need to really make sure that all possible entailed
58. formulas are inferred by the system, which
59. is more difficult to prove.
60. In propositional logic, there are two ways
61. to ensure completeness.
62. One is called proof by resolution.
63. And we're to use a powerful rule called resolution rule.
64. And the second one is called forward or backward chaining.
65. Then choose modus ponens on a restricted form of propositions
66. called horn clauses.
67. So modus ponen as is, that looks like p, p implies q infers q,
68. actually implies that we are using
69. a knowledge base that has an implication
70. of positive literals.
71. This is not always possible.
72. If the knowledge base is expressed
73. as an implication of positive literals,
74. then it's true that modus ponens is sound and complete.
75. But if we can't express knowledge base
76. as using this form, then it's not
77. possible to have a complete system with modus ponens.
78. So it's possible to use either resolution or backward
79. chaining on horn clauses to actually address
80. the problem of completeness.
81. So far, resolution is actually one single inference rule
82. that was invented by Robinson in 1965.
83. Resolution plus search is-- it turned out to be complete
84. and a sound inference algorithm.
85. I will now illustrate the concept of resolution
86. through the 1 plus word example.
87. We are in a setting in which the agent was in 11,
88. went to 21 to know that actually there is a breeze.
89. The agent didn't want to move forward to either 31 or 22
90. because there is a risk that it's a pit.
91. So it's a bottomless pit, so there is no risk to take here.
92. The agent decided to then go back to 11
93. and explore 12 to find that there actually
94. was a stench, but no breeze.
95. So we have the knowledge base initially.
96. So the knowledge base would be equal to the previous knowledge
97. and the new facts.
98. And these facts are, first of all,
99. that there is a breeze into 1.
100. And then if there is a breeze into 1,
101. we know that this means that there is either a pit in 11,
102. or a pit in 22, or a pit in 31.
103. So this will come and enrich the knowledge base.
104. When the agent came back to 11 and posted it to 21 too,
105. it felt a stench, in which case--
106. but no breeze.
107. So there is no breeze in the cell 12,
108. even though there is a breeze in 21.
109. And we could also actually infer that there
110. is no pit in 13, nor 22.
111. And the reason is that if there was a pit here or here,
112. then the agent would have felt a breeze here.
113. So there is no pit in 22, and there is no pit in 13.
114. So this new fact will come enrich the knowledge base.
115. We also have the fact that we got these two elements here
116. from the fact that if there is a breeze in 12,
117. then this means that there's either a pit in 13,
118. or a pit in 22, or a pit in 11.
119. So we have this new knowledge base now,
120. and we want to know whether we could infer from the knowledge
121. base, or we could infer that actually there is a pit in 31
122. or not.
123. So this is the question.
124. And it turned out that actually there is a pit there.
125. And the agent will infer that through some new inference
126. called resolution.
127. So the first thing to do is to look
128. at what's p31 actually appears.
129. And you see that p31 appears in b21.
130. So we have b21 if and only if p11
131. or p22 or p31, which we can write as a double implication.
132. So just to save space here, I'm going
133. to start right away by writing it as an implication.
134. B21 implies p11, or p22, or p31.
135. So I'm going to put an end in between.
136. And p11, or p22, or p31 implies b21.
137. So this is the first step from this element here.
138. I actually inferred this formula here
139. by just splitting the bi-directional
140. or the if and only if.
141. From this formula, given that I have the conjunction,
142. I'm going to pick one of the sites.
143. And this one, I pick this one, b21--
144. just strategy [INAUDIBLE], as I could get rid of the b21
145. because I know that in b21, b21 is true
146. based on the new information from the knowledge base.
147. B11, or p22, or p31-- sorry, 31.
148. And they have also that's b21 is true here.
149. From these two, by modus ponens, I can infer that I have--
150. so we have p.
151. I have p implies q, so that I have p11, or p22, or p31.
152. So there is a pit either in 11, or 22, or 31.
153. OK, great.
154. So now I know that, actually, there is no pit in 22
155. because if there was one, I would have felt a breeze here.
156. So from the fact that here there's no pit in 22,
157. this is where our resolution would come to work here.
158. So we have-- actually, there is no pit in 22,
159. and there is a pit in 22 here.
160. So these two would resolve with each other.
161. This means that there is either a pit in 11, or 22, or in 31.
162. But there is no pit in 22.
163. So it must be either in 11 or in 31.
164. So I'm going to make a nice interface here
165. by resolution in which these two elements here would resolve.
166. They resolve and make a new element called the resolvent.
167. And this resolvent is p11 or p31.
168. Then from this information, I know
169. that there is no pit in 11.
170. This is, by definition, the starting point of the puzzle.
171. So I know that there is no pit in 11.
172. I could apply, again, the resolution here between--
173. not between 11 and pit in 11.
174. So the resolution here would bring me to the fact
175. that, actually, p31, is actually a pit.
176. In other words, there is a pit in 31,
177. just because this is the resolvent of this 2,
178. and it is here.
179. If there is a pit in 11 or p31 and it is not p11,
180. then it must be 31.
181. So here I found--
182. I made the entailment that kb actually
183. is entailing that p31 is actually a pit--
184. and actually, this is decided by the Wumpus
185. based on this resolution inference.
186. We could do a similar work to find that actually there
187. is a Wumpus in 13.
188. So this resolution happened here at this stage
189. here and at that stage here.
190. And this element is called the resolvent.
191. And this element also is called the resolvent
192. of this 2 proposition here.
193. After this example, let's now talk more
194. formally about resolution.
195. Resolution is based on the concept of unit resolution,
196. in which we have an inference rule of the form l1, or l1,
197. or li, or dot, dot, dot, or lk, and the literal m resolving
198. to this formula here.
199. So specifically, li and m are called complimentary literals,
200. which means that one is the negation of the other one.
201. So if we do the unit resolution, it
202. means that you are going to get rid of the m
203. and of the li in the disjunction of the resolvent.
204. So these two, in other words--
205. these two elements here, the m and the li in the middle
206. here would resolve to obtain the resolvent that actually
207. is the disjunction of everything except the m and the li
208. because li and m are complimentary literals.
209. In other words, again, one is the negation of the other one.
210. Examples we have seen just right now,
211. if we have the negation of p22, and we have p13 or p22--
212. in other words, if there is a pit either in 13 or 22,
213. and there is no pit in 22 then we have only one possibility,
214. that the pit is in 13.
215. All right, so this is the principle
216. of resolution in general terms.
217. We couldn't close a disjunction o literals.
218. And a unit resolution is then applying a clause to a clause
219. and to a literal to obtain a new clause.
220. So this is the idea of resolution.
221. We have to have a disjunction of literals,
222. and we have to have a disjunction of literals
223. along with one literal in which this literal
224. along with the complimentary one in this clause
225. will resolve to obtain a new clause.
226. It is possible to generalize this unit resolution
227. into a more general form in which we have two clauses.
228. We have li or lk.
229. We also have m1 or mn.
230. And we have these two elements li
231. and mj that are complementary literals.
232. If one of them is the negation of the other one,
233. then we could resolve these two clauses
234. to the clause that has a disjunction of all
235. the literals except li and mj.
236. This is powerful, but it applies only on clauses.
237. And the question is now can we express
238. any proposition in propositional logic in terms of clauses?
239. And the answer is yes.
240. Every sentence in propositional logic
241. is actually logically equivalent to a conjunction
242. of clauses.
243. This is powerful.
244. And we call this form of representation
245. the conjunctive normal form, or CNF,
246. which is a conjunction of disjunction of literals,
247. in other words, a conjunction of clauses.
248. Here's an example of a conjunction
249. of disjunction of literals, in which we
250. have the conjunction of what?
251. Of this term on the left and this term on the right.
252. So we have a or not b get the disjunction of literals.
253. And this is a disjunction of the b or not c.
254. So this is an example of a propositional logic
255. form into a CNF.
256. And the other good news is that, actually,
257. if you use resolution inference, this
258. is a strong rule that would actually get rid
259. of all of the complementary elements
260. or literals in the clauses.
261. Then this form of resolution is actually sound, and compete,
262. and can be provable.
263. So it's possible to prove that we are not only
264. using a sound inference rule, because this make sense.
265. If you use the proof by truth table
266. of this kind of resolution, you find that it is sound.
267. And applying this resolution over and over again is sound.
268. And it's also complete.
269. And you can find the proof in the book
270. if you're interested about the completeness of the proof
271. by resolution.